

Genererende funktioner

Vi loader pakken **gfun**

```
> restart;with(gfun);
```

```
[Laplace, Parameters, algebraicsubs, algeqtodiffeq, algeqtoseries, algfuntoalgeq, borel,
cauchyproduct, diffeq*diffeq, diffeq+diffeq, diffeqtohomdiffeq, diffeqtorec, guesseqn,
guessgf, hadamardproduct, holexprtodiffeq, invborel, listtoalgeq, listtodiffeq,
listtohypergeom, listtolist, listtoratpoly, listtorec, listtoseries, poltodiffeq, poltorec,
ratpolytcoeff, rec*rec, rec+rec, rectodiffeq, rectohomrec, rectoproc, seriestoalgeq,
seriestodiffeq, seriestohypergeom, seriestolist, seriestoratpoly, seriestorec, seriestoseries]
```

(1)

Nogle lister

```
> L__1:=[1,-1,sqrt(2),27];
L__2:=[seq(1,n=0..20)];
L__3:=[seq(n,n=0..10)];
```

$$L_1 := [1, -1, \sqrt{2}, 27]$$

$$L_2 := [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]$$

$$L_3 := [0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10]$$

(2)

Kommandoer **listtoseries** hænger lister til tørre på potenserne

```
> listtoseries(L__1,x);
```

$$1 - x + \sqrt{2} x^2 + 27 x^3 + O(x^4)$$

(3)

```
> listtoseries(L__2,x);
```

$$1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9 + x^{10} + x^{11} + x^{12} + x^{13} + x^{14} + x^{15} + x^{16} + x^{17} + x^{18} + x^{19} + x^{20} + O(x^{21})$$

(4)

```
> listtoseries(L__3,x);
```

$$x + 2 x^2 + 3 x^3 + 4 x^4 + 5 x^5 + 6 x^6 + 7 x^7 + 8 x^8 + 9 x^9 + 10 x^{10} + O(x^{11})$$

(5)

Kommandoer **series** udregner generende funktioner på standardformen som et polynomium af i princippet uendelig grad, fx

```
> series((4)+(5),x,20);
```

```
series((4)*(5),x,20);
```

$$1 + 2 x + 3 x^2 + 4 x^3 + 5 x^4 + 6 x^5 + 7 x^6 + 8 x^7 + 9 x^8 + 10 x^9 + 11 x^{10} + O(x^{11})$$

$$x + 3 x^2 + 6 x^3 + 10 x^4 + 15 x^5 + 21 x^6 + 28 x^7 + 36 x^8 + 45 x^9 + 55 x^{10} + O(x^{11})$$

(6)

```
> series(1/(1-x),x,20);
```

```
series(1/(1-a*x),x,10);
```

$$1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9 + x^{10} + x^{11} + x^{12} + x^{13} + x^{14} + x^{15} + x^{16} + x^{17} + x^{18} + x^{19} + O(x^{20})$$

$$1 + a x + a^2 x^2 + a^3 x^3 + a^4 x^4 + a^5 x^5 + a^6 x^6 + a^7 x^7 + a^8 x^8 + a^9 x^9 + O(x^{10})$$

(7)

Bemærk syntaksen **series(genererende funktion, variabel, højeste grad man regner til)**

Kommandoer **seriestolist** giver den liste som en genererende funktion frembringer

```
> seriestolist((6)) ;
```

$$[0, 1, 3, 6, 10, 15, 21, 28, 36, 45, 55]$$
(8)

Man kan være interesseret i et enkelt element fra listen

```
> seriestolist((6)) [8] ;
```

$$28$$
(9)

De generelle regneregler for genererende funktioner

```
> L:= [seq(a[i], i=0..10)] ; K:= [seq(b[i], i=0..10)] ;
```

$$L := [a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}]$$

$$K := [b_0, b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9, b_{10}]$$
(1.1)

```
> A:=listtoseries(L,x) ; B:=listtoseries(K,x) ;
```

$$A := a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 + a_6 x^6 + a_7 x^7 + a_8 x^8 + a_9 x^9 + a_{10} x^{10} + O(x^{11})$$

$$B := b_0 + b_1 x + b_2 x^2 + b_3 x^3 + b_4 x^4 + b_5 x^5 + b_6 x^6 + b_7 x^7 + b_8 x^8 + b_9 x^9 + b_{10} x^{10} + O(x^{11})$$
(1.2)

```
> series(3*A,x,25) ;
```

$$3 a_0 + 3 a_1 x + 3 a_2 x^2 + 3 a_3 x^3 + 3 a_4 x^4 + 3 a_5 x^5 + 3 a_6 x^6 + 3 a_7 x^7 + 3 a_8 x^8 + 3 a_9 x^9 + 3 a_{10} x^{10} + O(x^{11})$$
(1.3)

```
> series(A+B,x,25) ;
series(A-B,x,25) ;
```

$$a_0 + b_0 + (a_1 + b_1) x + (a_2 + b_2) x^2 + (a_3 + b_3) x^3 + (a_4 + b_4) x^4 + (a_5 + b_5) x^5 + (a_6 + b_6) x^6 + (a_7 + b_7) x^7 + (a_8 + b_8) x^8 + (a_9 + b_9) x^9 + (a_{10} + b_{10}) x^{10} + O(x^{11})$$

$$a_0 - b_0 + (a_1 - b_1) x + (a_2 - b_2) x^2 + (a_3 - b_3) x^3 + (a_4 - b_4) x^4 + (a_5 - b_5) x^5 + (a_6 - b_6) x^6 + (a_7 - b_7) x^7 + (a_8 - b_8) x^8 + (a_9 - b_9) x^9 + (a_{10} - b_{10}) x^{10} + O(x^{11})$$
(1.4)

```
> series(A*B,x,25) ;
```

$$a_0 b_0 + (a_0 b_1 + a_1 b_0) x + (a_0 b_2 + a_1 b_1 + a_2 b_0) x^2 + (a_0 b_3 + a_1 b_2 + a_2 b_1 + a_3 b_0) x^3 + (a_0 b_4 + a_1 b_3 + a_2 b_2 + a_3 b_1 + a_4 b_0) x^4 + (a_0 b_5 + a_1 b_4 + a_2 b_3 + a_3 b_2 + a_4 b_1 + a_5 b_0) x^5 + (a_0 b_6 + a_1 b_5 + a_2 b_4 + a_3 b_3 + a_4 b_2 + a_5 b_1 + a_6 b_0) x^6 + (a_0 b_7 + a_1 b_6 + a_2 b_5 + a_3 b_4 + a_4 b_3 + a_5 b_2 + a_6 b_1 + a_7 b_0) x^7 + (a_0 b_8 + a_1 b_7 + a_2 b_6 + a_3 b_5 + a_4 b_4 + a_5 b_3 + a_6 b_2 + a_7 b_1 + a_8 b_0) x^8 + (a_0 b_9 + a_1 b_8 + a_2 b_7 + a_3 b_6 + a_4 b_5 + a_5 b_4 + a_6 b_3 + a_7 b_2 + a_8 b_1 + a_9 b_0) x^9 + (a_0 b_{10} + a_1 b_9 + a_2 b_8 + a_3 b_7 + a_4 b_6 + a_5 b_5 + a_6 b_4 + a_7 b_3 + a_8 b_2 + a_9 b_1 + a_{10} b_0) x^{10} + O(x^{11})$$
(1.5)

► Fibonacci

► **Hæveautomat**

► **Annuitet**